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CS 230

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Short Assignment 3

(define multiply

(lambda ((a <number>) (b <integer>))

(cond ((zero? b) 0)

((odd? b)

(+ a (multiply (+ a a) (quotient b 2))))

(else

(multiply (+ a a) (quotient b 2))))))

Proof by induction on variable b, natural number

P(b): (multiply a b) = a\*b

Base Case:

Prove that (multiply a {0}) = a\*0

(multiply a {0}) by the substitution model is

(cond ((zero? {0}) 0 …)

which is {0}

and that is right as a\*0 = 0

Inductive Hypothesis:

For all k, elements of the set of all natural numbers, less than b, assume that (multiply a k) = a\*k

Inductive Step:

Show that (multiply a b) = a\*b

cond ((zero? {b}) 0)

((odd? {b})

(+ {a} (multiply (+ {a} {a}) (quotient {b} 2))))

(else

(multiply (+ {a} {a}) (quotient {b} 2))))))

{b} can’t be 0 as k is a natural number and less than b

if (b) is odd,

(+ {a} (multiply (+ {a} {a}) (quotient {b} 2)))

We know that (+ {a} {a}) is {2a}

Since b is odd, (quotient {b} 2) is {b/2 – 1/2} which is less than k

So we can simplify to (+ {a} (multiply {2a} {b/2 – 1/2}) and because b/2 is less than b, by the inductive hypothesis,

(multiply {2a} {b/2 – 1/2}) = {2a} \* {b/2 – 1/2} = a\*(b – 1) = a\*b – a

Plugging that into the formula,

(+ {a} (multiply (+ {a} {a}) (quotient {b} 2))) = (+ {a} {a\*b – a}) = a\*b

if (b) is even,

(multiply (+ {a} {a}) (quotient {b} 2)))

We know that (+ {a} {a}) is {2a} and

Because b is even, (quotient {b} 2) is {b/2} which is less than k

So we can simplify to (multiply {2a} {b/2}) and because b/2 is less than b, by the inductive

hypothesis,

(multiply (+ {a} {a}) (quotient {b} 2))) = (multiply {2a} {b/2} = {2a} \* {b/2}

= a\*b

Thus by induction, (multiply a b) = a\*b